Thermal Instability of Non-Newtonian Fluid with uniform Magnetic field in a NON-Rotating Medium

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Abstract

The thermal instability of a Non-Newtonian fluid in the presence of uniform magnetic field in a non-rotating medium is considered. For the case of stationary convection, fluid behaves like a Newtonian fluid. It is found that the magnetic field has both stabilizing and destabilizing effects.

Keywords: Non-Newtonian fluid, thermal instability, magnetic field

1. Introduction

The thermal instability of a fluid layer with maintained adverse temperature gradient by heating the underside, play an important role in geophysics, interior of the earth, oceanography and atmospheric physics etc. a detailed account of the theoretical and experimental study of the onset of Benard convection in Newtonian fluids. Under varying assumptions of hydrodynamics, has been given by Chandrasekhar [1]. The thermal instability of a Maxwell fluid in hydromagnetics has been studied by Bhatia and Steiner [2]. They have found that the magnetic field stabilizes a viscoelastic fluid just as the Newtonian fluid. Effect of magnetic field on thermal instability of a Rotating Rivlin – Ericksen Viscoelastic fluid has been studied by pardeep kumar and Hari Mohan [3]. The use of Boussinesq approximation has been made throughout, which states that the density may be treated as a constant in all the terms in the equations of motion except the external force term. Sekar. R *et al.* [4] have studied Ferro convection in an anisotropic porous medium. Here the magnetic field has stabilizing effects on the stationary convection and introduce Oscillatory modes in the system. Goel. A. K, Agrawal. S. C [5] have studied a Numerical study of hydromagnetic thermal convection in a visco- elastic density fluid in a Porous Medium. Chen.H and Chen.C [6] have studied free convection flow of Non – Newtonian embedded in a porous medium. The effect of uniform magnetic fluid on thermal instability of Non – Newtonian fluid in an anisotropic porous medium. I.G. Oldroyd [8] have studied the non –Newtonian effects in steady motion of some idealized elastico –viscous liquid. Anoj Kumar, B.S. Bhadauria [9] have investicated the thermal instability in a rotating anisotropic porous layer saturated by a viscoelastic fluid.

2. Mathematical Formulation of the Problem and Peturbation Equations

Consider an infinite, horizontal in compressible Non Newtonian fluid layer of thickness *d*, heated from below , so that , the temperature and density at the bottom surface *z=0* are T_0, ρ_0 respectively and at the upper surface $z = d$ are T_d, ρ_d and that a uniform adverse temperature gradient $\beta = (|\, dT / dz\,|)$ is maintained. Let ρ , *p, T* and $\vec{q}(u,v,w)$ denote respectively the density pressure, temperature and velocity of the fluid , $\vec{q}(x,t)$ and $N(x,t)$ denote the velocity and number density of suspended particles respectively.

The equation of motion is

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}.\nabla)\vec{v} = -\nabla\left(\frac{p}{\rho_0}\right) + \vec{g}\left(1 + \frac{\delta p}{\rho_0}\right) + \left(\upsilon + \upsilon'\frac{\delta}{\delta t}\right)\nabla^2\vec{v} + \frac{\mu_e}{4\pi\rho_0}(\nabla\times\vec{H})\times\vec{H} \tag{2.1}$$

Equation of continuity is

$$\nabla.\vec{v} = 0 \tag{2.2}$$

Heat conduction and Maxwell's equation are

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$$\frac{\partial T}{\partial t} + (\vec{v}.\nabla)T = \chi \nabla^2 T \qquad (2.3)$$

$$\nabla.\vec{H} = 0 \qquad (2.4)$$

$$\frac{\partial \vec{H}}{\partial t} = (\vec{H}.\nabla)\vec{v} = \eta \nabla^2 \vec{H} \qquad (2.5)$$

where $\vec{v}(u,v,w)$, P, ρ, T, υ and υ' denote the velocity, pressure, density, temperature, kinematic viscosity and kinematic viscoelasticity respectively and $\vec{r}(u,v,w)$.

The equation of state for the fluid is

$$\rho = \rho_0[1 - \alpha(T - T_0)] \qquad (2.6)$$

where ρ_o, T_o are respectively the density and temperature of the fluid at the reference level z = 0 and α is the co-efficient of thermal expansion.

The initial state is one in which the velocity, density, pressure and temperature at any point in the fluid are respectively, given by

$$\vec{v} = (0,0,0),\ \rho = \rho(z),\ p = p(z),\ T = T(z) \qquad (2.7)$$

The change in density δρ, caused by the perturbation θ in temperature is given by

$$\rho + \delta\rho = \rho_0[1 - \alpha(T + \theta - T_0)] = \rho - \alpha\rho_0\theta$$

i.e., $\delta\rho = -\alpha\rho_0\theta$ (2.8)

Then the linearised perturbation equations are

$$\frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho_0}\nabla(\delta p) + \vec{g}(\alpha\theta) + \left(\upsilon + \upsilon'\frac{\delta}{\delta t}\right)\nabla^2\vec{v} + \frac{\mu_e}{4\pi\rho_0}(\nabla\times\vec{h})\times\vec{H} \qquad (2.9)$$

$$\nabla.\vec{v} = 0 \qquad (2.10)$$

$$\frac{\partial \theta}{\partial t} = \beta w + \chi\nabla^2\theta \qquad (2.11)$$

$$\nabla.\vec{h} = 0 \qquad (2.12)$$

$$\frac{\partial \vec{h}}{\partial t} = (\vec{H}.\nabla)\vec{v} = \eta\nabla^2\vec{h} \qquad (2.13)$$

The perturbation quantities in the normal mode are taken as

$[w,\theta, h,\xi]=[w(z),\theta(z),K(z),\chi(z)]exp\ (i\ k_x x+ik_y y+nt)$

where k_x,k_y are the wave numbers along x and y directions respectively.

$$\left[\sigma(D^2 \quad a^2)W + \left(\frac{g\alpha d^2}{v}\right)a^2\Theta \quad \frac{\mu_e H d}{4\pi\rho_0 v}(D^2 \quad a^2)DK\right] = [1 + F\sigma](D^2 \quad a^2)^2 W \qquad (2.14)$$

$$[\ D^2 - a^2 - \rho_1\sigma\ ]\Theta = -\left(\frac{\beta d^2}{x}\right)W \qquad (2.15)$$

$$[\ D^2 - a^2 - \rho_1\sigma\ ]K = -\left(\frac{Hd}{\eta}\right)DW \qquad (2.16)$$

Free – Free boundary conditions are

$W = D^2W = 0,\ \theta = 0$ at $z=0, z=1,$

$DX=0,\ K=0$ (2.14)

On a perfectly conducting boundary. We obtain the dispersion relation

$$R_1 = \left(\frac{1+x}{x}\right)\frac{[\{(1+iF_1\sigma_1\pi^2)(1+X)+i\sigma_1\}\{1+x+i\sigma_1 p_1\}+ (}{(1+x+i\sigma_1 p_1)}$$

$$\frac{[1+x+i\sigma_1 p_1]}{(1+x+i\sigma_1 p_2)} +$$

$$\frac{T_1\ (1+x+i\sigma_1 p_1)(1+x+i\sigma_1 p_1)}{\{x[\{(1+iF_1\sigma_1\pi^2)(1+x)+i\sigma_1\}\{1+x+i\sigma_1 p_2\}+Q_1]]} \qquad (2.15)$$

3. The Stationary Convection

When the instability sets in as stationary convection, the marginal state will be characterized by σ = 0. The dispersion relation is

$$R_1 = \left(\frac{1+x}{x}\right)[(1+x)^2 + Q_1] \qquad (3.1)$$

a result given by Chandrasekaran [1].

Thus we have for the stationary convection, the visco-elasticity parameter F vanishes with σ and non-newtonian fluid behaves like an ordinary Newtonian fluid. To study the effects of rotation and magnetic field, we examine the nature of and dR_1 / dQ_1 .

Equation (3.1) yields

$$\frac{dR_1}{dQ_1} = \frac{1+X}{X} \qquad (3.2)$$

It is also clear from (3.2) that for a stationary convection dR_1 / dQ_1 may be positive as well as negative. Thus the magnetic field has both stabilizing and destabilizing effects on the system.

The variation of R_1 with Q_1 for fixed value of x=3,4 is represented in Table 1 and the Fig. 1 shows the variation of R_1 with respect to Q_1. It clearly depicts both the stabilizing and destabilizing effects of the magnetic filled on the system.

Table. 1
The variation of R_1 with Q_1 for fixed value of x = 3 ,4

Sl.No.	**Q_1**	**R_1**	
		X=3	**X=4**
1.	1	22.67	32.50
2.	4	26.67	36.25
3.	7	30.67	40.00
4.	10	34.67	43.75
5.	13	38.67	47.50
6.	16	42.67	51.25
7.	19	46.67	55.00

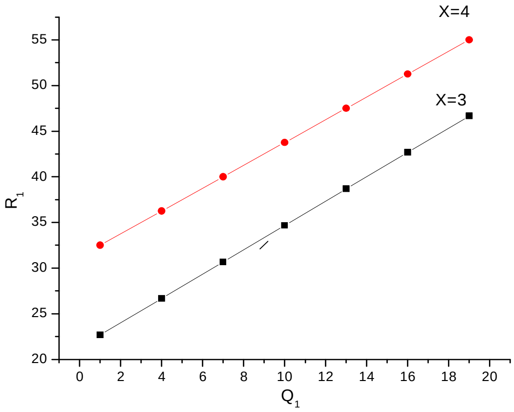

Fig. 1. Variation of R_1 with Q_1 for fixed value of x = 3 ,4

4. Results and Discussion

The thermal instability of Non – Newtonian fluid with uniform magnetic field in a non –rotating medium has been analyzed using dispersion relation. The critical magnetic thermal Rayleigh number increases with different value of Q_1. From the above discussion and analysis one can conclude that the magnetic field has both stabilizing and destabilizing effects on the system.

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